SEMIDEFINITE APPROXIMATIONS OF THE POLYNOMIAL ABSCISSA

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This talk reports on some results of our joint work [3]. The abscissa of a univariate parameterized polynomial is the maximal real part of its roots [1]. It occurs, for example, when studying linear differential equations as a measure of the decay or growth rate of the solution. Furthermore, in the space of controller parameters one is interested in the zero sublevel set of the abscissa, called the stability region [2].

The abscissa function is continuous, but in general not locally Lipschitz. This low regularity causes numerical difficulties when designing and optimizing control laws and therefore motivates upper and lower approximations of the abscissa by less complex functions, in our case polynomials of fixed degree.

For the approximation from above we rewrite the set of zeros as an appropriate basic closed semi-algebraic set and construct an infinite dimensional linear programming problem whose solution is the abscissa. Afterwards we build a hierarchy of finite dimensional semidefinite programming problems by replacing the nonnegativity constraint by a specific certificate of positivity as described in [4]. We prove that the resulting sequence of polynomials of increasing degree converges to the abscissa in L_1-norm.

Constructing a lower bound on the abscissa proved to be much more challenging. We propose two methods: first, a direct one using elementary symmetric functions which is very neat and general, but involves many variables, and second, an approach using the Gauß-Lucas theorem which is computationally faster, but more complicated and subject to assumptions.

Références

[1] J.A. Cross, *Spectral abscissa optimization using polynomial stability conditions.* PhD thesis, University of Washington, SEATTLE, 2010.

[2] D. Henrion, J.B. Lasserre, *Inner approximations for polynomial matrix inequalities and robust stability regions.* IEEE Transactions on Automatic Control 57(6):1456-1467, 2012.

[3] R. Hess, D. Henrion, J.B. Lasserre, T.-S. Pham, *Semidefinite approximations of the polynomial abscissa.* https://hal.archives-ouvertes.fr/hal-01181623

[4] J.B. Lasserre, *Moments, positive polynomials and their applications.* IMPERIAL COLLEGE PRESS, LONDON, 2010.