Chance constrained optimization of a three-stage launcher

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We are interested in optimizing the fuel load of a three-stage launcher which mission is to deliver a payload to a given altitude. Since some of the system’s parameters are subject to uncertainties, the aim is to guarantee the success of the mission with a 90% probability, taking into account those random variations.

A general formulation of our stochastic optimization problem would be

\[
\begin{align*}
\text{Calculate} & \quad \min_{x \in X} J(x) \\
\text{Subject to} & \quad P[G(x, \omega) \leq 0] \geq p
\end{align*}
\]

where \( x \in X \subseteq \mathbb{R}^n \) is the array of optimization variables, \( \omega \in \Omega \subseteq \mathbb{R}^n \) is an array of random variable with known distribution and \( 0 \leq p \leq 1 \) represents the probability of success. Our approach consists in translating this stochastic optimization problem into a deterministic one by approximating the distribution of the constraint function \( G \) (which is a random variable, since it depends on \( \omega \)) via Kernel Density Estimation. Let \( s \) be a random variable with an unknown probability density function \( f \) that we want to estimate and let \( \{s_1, s_2, \ldots, s_n\} \) be a sample of size \( m \) from the variable \( s \). A Kernel Density Estimator for \( f \) is the function

\[
\hat{f}(x) := \frac{1}{m h} \sum_{i=1}^{m} K \left( \frac{x - s_i}{h} \right)
\]

where the function \( K \) is called kernel and the smoothing parameter \( h \) is called bandwidth. The approximation error between \( f \) and \( \hat{f} \) depends on the choice of both \( K \) and \( h \). Since the bandwidth plays a much more important role than the kernel (see [1]), in most applications the study is focused on the choice of \( h \). \( K \) is usually the Gaussian kernel.

References