# Chance constrained optimization of a three-stage launcher

**Achille Sassi**
ENSTA ParisTech, France

**Jean-Baptiste Caillau**
Université de Bourgogne, France

**Max Cerf**
Airbus Defence and Space, France

**Emmanuel Trélat**
Université Pierre et Marie Curie, France

**Hasnaa Zidani**
ENSTA ParisTech, France

**Keywords :** Stochastic optimization, density estimation, chance constraint

We are interested in optimizing the fuel load of a three-stage launcher which mission is to deliver a payload to a given altitude. Since some of the system's parameters are subject to uncertainties, the aim is to guarantee the success of the mission with a 90% probability, taking into account those random variations.
A general formulation of our stochastic optimization problem would be

$$\begin{cases} \text{Calculate} \\ \quad \min_{x \in X} J(x) \\ \text{Subject to} \\ \quad \mathbb{P}\big[G(x,\omega) \le 0\big] \ge p \end{cases}$$

where $x \in X \subset \mathbb{R}^n$ is the array of optimization variables, $\omega \in \Omega \subset \mathbb{R}^n$ is an array of random variable with known distribution and $0 \le p \le 1$ represents the probability of success. Our approach consists in translating this stochastic optimization problem into a deterministic one by approximating the distribution of the constraint function $G$ (which is a random variable, since it depends on $\omega$) via Kernel Density Estimation. Let $s$ be a random variable with an unknown probability density function $f$ that we want to estimate and let $\{s_1, s_2, \dots, s_n\}$ be a sample of size $m$ from the variable $s$. A Kernel Density Estimator for $f$ is the function

$$\hat{f}(x) := \frac{1}{mh} \sum_{i=1}^{m} K\left(\frac{x - s_i}{h}\right)$$

where the function $K$ is called *kernel* and the smoothing parameter $h$ is called *bandwidth*. The approximation error between $f$ and $\hat{f}$ depends on the choice of both $K$ and $h$. Since the bandwidth plays a much more important role than the kernel (see [1]), in most applications the study is focused on the choice of $h$. $K$ is usually the Gaussian kernel.

## References

[1] S. J. Sheather, *Density Estimation*, Statistical Science 19(4), 588-597, 2004.