L^1-Optimality Conditions for Circular Restricted Three-Body Problem

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As an increasing number of artificial satellites or spacecrafts have been and are being launched into deeper space since 1960s, the problem of controlling the translational motion of a spacecraft in the gravitational field of multiple celestial bodies such that some cost functionals are minimized or maximized arises in astronautics. The circular restricted three-body problem (CRTBP), which though as a degenerate model in celestial mechanics can capture the chaotic property of n-body problem, is extensively used in the literature in recent years to study optimal trajectories in deeper space. We consider the L^1-minimization problem for the translational motion of a spacecraft in a CRTBP, which aims at minimizing the L^1-norm of control. Necessary conditions are derived by using the Pontryagin Maximum Principle, revealing the existence of bang-bang and singular controls. Singular extremals are detailed, recalling the existence of the Fuller phenomena according to the theories developed by Marchal in Ref. [1] and Zelikin *et al.* in Refs. [2, 3]. Then, according to the theory of field of extremals in Refs. [5, 4], some sufficient optimality conditions for the nonsingular extremals with bang-bang controls are established. Finally, employing a combination of a shooting method with a continuation method of Caillau *et al.* in Refs. [6, 7], an L^1-minimization trajectory for the translational motion of the spacecraft is computed and the local optimality of the computed trajectory is checked thanks to the optimality conditions established. This is an adjoint work with J.-B. Caillau (Université de Bourgogne) and Y. Chitour (Univesity Paris-Sud)

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