An alternative class of distortion operators: an alternative tool to generate asymmetrical multimodal distribution

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To integrate financial and actuarial insurance pricing theories, Wang (2000) proposes a form of insurance risk pricing based on the standard Gaussian cumulative distribution function (cdf) distortion operator with one parameter. He points out that this operator is either concave or convex. A number of papers propose applications of distortion operators. For example, Lin and Cox (2005) successfully applies Wang transform to price mortality risk bonds. De Jong and Marshall (2007) provides a method for analysing and projecting mortality based on the Wang Transform. Denuit et al. (2007) designs the survivor bonds with the help of Wang transform (Wang (2000)) which could be issued directly by insurers. However, Godin et al. (2012) argues that it is a well-known fact that the returns of most financial assets have semi-heavy tails. Consequently he recognizes that a downside of the normal distortion of Wang (2000) is its underlying symmetrical that poses some constraints in applications. More precisely, Guégan and Hassani (2015) criticize that Wang (2000) applies the same perspective of preference to quantify the risk associated to gain and risk. Thus, the risk manager evaluates the risk associated to the upside and downside risks with the same function that implies a symmetrical consideration for the two effects due to the distortion. Accordingly, a number of papers make proposals on how to avoid the problem of symmetry in the previous distortion operators. For example, van der Hoek and Sherris (2001), Wang (2004), Sereda et al. (2010), Godin (2012) and Guégan and Hassani (2015) et al.. Importantly, motivated by the crisis, the multimodal characteristic of distributions of some economic variables, for example some stock market indexes like S&P 500 index (the Standard & Poor's 500), SHCOMP index (Shanghai Stock Exchange Composite Index) and FTSE Index (the Financial Time Stock Exchange 100 Index), can include useful information of systemic risk. Accordingly it is necessary to find a model such that it is flexible enough to accommodate various shapes of continuous distributions with leptokurtic, skewed and multimodal characteristics. Additionally, the financial industry has extensively used quantile-based downside risk measures based on the Value-at-Risk (VaR). In actuarial terms, VaR is a quantile reserve, often using the pth percentile of the loss distribution. We should emphasize that when we compute the VaR, the explicit analytical form of the inverse mapping of cdf is crucial. Consequently, we propose an alternative class of distortion operators allowing to build an asymmetrical multimodal distribution, with explicit analytical inverse mapping.

Références

[1] Wang SS (2000) A Class of Distortion Operators for Pricing Financial and Insurance Risks. The Journal of Risk and Insurance 67: 15-36