Regularization via sets satisfying the interior sphere condition

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For a given closed set $S \subset \mathbb{R}^n$, we provide inner approximation of S by sets satisfying the *interior sphere condition*. The fact that our approximation sets satisfy the interior sphere condition with variable radius, allows us to approach any corner and to use the Pompeiu-Hausdroff convergence even if the set S is unbounded. Our results reached relate to results of Clarke, Ledyayev and Stern [1]. In this reference, the authors provided an inner approximation of a given *wedged* (or *epi-Lipschitz*) set S by φ_0-*convex* (or *proximally smooth*) sets. More precisely, Clarke, Ledyayev and Stern proved in [1] that if $S \subset \mathbb{R}^n$ is a nonempty, compact and wedged set then the *r-inner approximation* S_r is φ_0-convex, for r sufficiently small, with φ_0 depending on r (recall that S_r is the set of all points in S having a distance to S^c, the complement of S, greater than or equal to r). Some improvements of this result can be found in the works of Cornet and Czarnecki [2, 3]. Our main result differs from these results by several features. The main ones are that we do not make such wedgedness assumption on S and our approximation sets satisfy an interior sphere condition which is a different regularity property from the φ_0-convexity as is already shown in Nour, Stern and Takche [4].

Références

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