Extended Linear Quadratic Programming Applied to Estimation and Control

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Extended Linear Quadratic Programming, first introduced by Rockafellar and Wets in [1], is an elegant framework that contains many classical optimization problems. An optimization problem is said to be a *Linear Quadratic Program* if it is of the form

$$\min_{u \in U} f(u) = \sup_{v \in V} J(u, v)$$

where $J(u, v)$ is a convex-concave linear-quadratic function and both U and V are nonempty polyhedral subsets of $\mathbb{R}^n$. Linear Quadratic Programs (LQP) include many multistage problems from stochastic programming in addtion to models from stochastic control.

The linear estimation problem, in which the goal is to find the best estimate of the state of a linear dynamical system given a set of noisy measurements, fits directly into the linear quadratic programming framework. Exploiting the simplicity of the model, we verify the classical duality result between linear estimation and control. By using an equivalent Dynamic Programming formulation, we show the duality correspondence between linear stochastic filtering and the deterministic Linear-Quadratic Regulator problem of optimal control.

In addition to providing an elegant framework to verify classical results, formulating estimation as a LQP allows us to extend the estimation problem to other useful settings. By using monitoring functions in our problem formulation, we can extend the typical mean-squared error estimation problem to more general penalities. For example, using the L_1 norm allows more robust estimation in the presence of outlier measurements. Formulating the corresponding dual problem then provides further insight into the depth of the relationship between estimation and control.

We also use the stochastic programming framework to generalize the estimation problem to the setting where measurements are omitted or arrive out of order. As an example, we consider the problem of estimating the location of an unmanned vehicle given noisy GPS measurements. By using the splitting algorithms of Korf [3] and [2] we solve the primal and dual problems simultaneously. Lastly, we interpret the dual variables in order to assess the value of relaxing the condition that our estimation depends only on the observed measurements.

Références

[1] R. T. Rockafellar, R. Wets, “Generalized Linear-Quadratic Problems of Deterministic and Optimal Control in Discrete Time” *SIAM Journal of Control and Optimization*, 28, No. 4 (1990), 810-822.

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