A new direction in polynomial time interior-point methods for monotone linear complementarity problem

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The monotone linear complementarity problem (LCP) is defined by square matrices $M, N \in \mathbb{R}^{n\times n}$ and a vector $q \in \mathbb{R}^n$, where M and N satisfy a monotonicity property : all vectors $y \in \mathbb{R}^n$ and $z \in \mathbb{R}^n$ that satisfy $My + Nz = 0$ have $y^T z \geq 0$. This problem consists in finding vectors y and z such that

$$My + Nz = q,\ (y, z) \geq 0,\ y^T z = 0\ . \tag{1}$$

The main idea of interior-point methods is to follow the central path defined for $\mu \geq 0$ by

$$\begin{array}{l} My + Nz = q \\ (y, z) \geq 0 \\ y \circ z = e\mu \end{array}\ , \tag{2}$$

where $\circ$ denotes the componentwise Hadamard product and $e \in \mathbb{R}^n$ is the vector with $e_i = 1,\ \forall i = 1, ..., n$. These methods have received a wide interest thanks to their strong theoretical complexity. Actually for an appropriate starting point $n\mu^0 = (y^0)^T z^0$ complexity of the interior-point methods to a point with $y^T z \leq \epsilon$ is

$$O(n^\tau \log(\frac{\mu^0}{\epsilon}))\text{ iterations,} \tag{3}$$

where $\tau = \frac{1}{2},\ 1$ or 2 depending on the algorithm.
In this paper we study a new way to find directions following the central path. Given concave C^2 functions $\varphi : \mathbb{R}^n_+ \to \mathbb{R}^n_+$, with $\varphi(0) = 0$ and $\varphi'(t) > 0$ for all $t \geq 0$, we consider for $\nu \geq 0$

$$\begin{array}{l} My + Nz = q \\ (y, z) \geq 0 \\ \varphi(y \circ z) = e\nu \end{array}\ . \tag{4}$$

In [1] Darvay introduces this new direction for $\varphi(t) = \sqrt{t}$. We generalize the proof for a family of concave functions and show that this new direction gives polynomial time complexity of the same order of existing interior-point methods. We will also discuss an infeasible version of the algorithm, which does not require to start on the central path, a comparison on the number of iterations with classical methods and the implementation of the algorithm. Moreover, most of the computational effort in these methods is taken up in solving linear systems and this new direction reduces the condition number of the linear systems.

Références

[1] Z. Darvay *Studia Universitatis Babes-Bolyai, Series Informatica. A weighted-path-following method for linear optimization*, Citeseer, 2002.