A Semi-Lagrangian scheme for a modified version of the Hughes model for pedestrian flow

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The interest for the study of pedestrians dynamics raised constantly in last decades. Coming from different sectors of the applied physics and transportation research this subject imposed itself to the attention of the applied mathematical community. The range of the models proposed for those phenomena goes from microscopic, where each individual is seen as a particle and its dynamics described separately, to macroscopic, where a crowd is seen as a distribution of density with common goals and rules of interaction.
In 2002 R. Hughes proposed a macropscopic model for pedestrian dynamics in [2] coupling the continuity equation (describing the evolution of the crowd density) with an Eikonal equation, which gives the shortest (possibly weighted) distance to an exit. Formally

$$\begin{cases} m_t - \mathrm{div}(m\, f^2(m)\nabla u) = 0, \\ |\nabla u| = \dfrac{1}{f(m)}, \end{cases}$$

where $x \in \Omega$ denotes the position in space, $t \in (0, T]$, $T \in \mathbb{R}^+$ represents the time and the ∇ operator represents the gradient with respect to the space variable x. The function $m = m(x, t)$ corresponds to the pedestrian density, the function u the weighted shortest distance to a target.
In this talk we propose a numerical scheme for a modified version of the Hughes model. This scheme can be seen as a semi-Lagrangian explicit approximation of a general non linear Fokker-Planck equation. In the model examined the pedestrians, represented by a density function, move on a bounded domain choosing a trajectory to minimize the instantaneous travel cost to the destination. The difference with the original model is the presence of diffusion in the process and a technical regularization of the function which penalizes the areas of high density. The scheme is shown to be effective to supply a good approximation of the process, easy to implement and well adapt to catch some essential features of the phenomena modeled.

Références

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