Lipschitz continuity of solutions for the Hamilton-Jacobi equations.

Thi Tuyen NGUYEN
Université de Rennes 1, France

Emmanuel CHASSEIGNE
Université de Tours, France

Olivier LEY
INSA de Rennes, France

Mots-clefs : Viscosity solutions, ellipticity, Ornstein-Uhlenbeck operator, non-local operator.

We study Lipschitz continuity of viscosity solutions of the stationary problem for the Hamilton-Jacobi equation

$$\lambda u^\lambda(x) - \mathrm{tr}(\sigma(x)\sigma^T(x)D^2u^\lambda(x)) + \langle b(x), Du^\lambda(x)\rangle + H(x, Du^\lambda(x)) = f(x) \quad \text{in } I\!R^N, \quad \lambda \in (0,1),$$

where σ is uniformly invertible, bounded and Lipschitz matrix, b is dissipative which is also called Ornstein-Uhlenbeck operator, f is locally Lipschitz and H satisfies

$$|H(x,p)| \leq C(1+|p|), \quad x, p \in I\!R^N.$$

We generalize the result obtained by Ishii, Fujita and Loreti [1]. The important idea here is to use the ellipticity coming from the second order term combined with the Ornstein-Uhlenbeck operator to behave the other terms of the equation.
Replacing the second order terms by the non-local term defined by

$$\mathcal{I}(x, u^\lambda, Du^\lambda) = \int_{I\!R^N} u^\lambda(x+z) - u^\lambda(x) - Du^\lambda(x).z\mathbb{I}_B(z)\nu(dz), \quad \nu(dz) = \frac{e^{-|z|}}{|z|^{N+\beta}}dz, \quad \beta \in (1,2),$$

we obtain also the Lipschitz result with an additional condition on the Ornstein-Uhlenbeck operator, since in this case the non-local term does not seem powerful enough to use ellipticity as in the local case.

Références

[1] Y. Fujita, H. Ishii, and P. Loreti, *Comm. Partial Differential Equations. Asymptotic solutions of viscous Hamilton-Jacobi equations with Ornstein-Uhlenbeck operator.*, 31(4-6):827–848, 2006.