

SEMIDEFINITE HIERARCHIES FOR POLYNOMIAL OPTIMIZATION

Monique LAURENT

CWI, Amsterdam, and Tilburg University

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Minimizing a polynomial function f over a region K defined by polynomial inequalities models broad classes of hard problems from combinatorics, geometry and optimization. New algorithmic approaches have emerged recently for computing the global minimum, by combining tools from real algebra (sums of squares of polynomials) and functional analysis (moments of measures) with semidefinite optimization. Sums of squares are used to certify positive polynomials and the moment approach gives tools to certify global optimality and extract global minimizers.

Various hierarchies of relaxations have been proposed in the recent years, in particular by Lasserre and Parrilo (see [1, 4]). These hierarchies exploit sums of squares representations of positive polynomials and the dual theory of moments. Roughly speaking, one can get lower bounds for the minimum of f by finding suitable sums of squares representations of f and one can find upper bounds by using probability measures on the set K with suitable density functions (like being a sum of squares).

We will review some properties of these hierarchies, with a special focus on some recent results about the convergence analysis of some of them. We will consider, in particular, polynomial optimization over the simplex and the hypercube, and a measure based hierarchy of upper bounds for optimization over a compact body K (see [2, 3]).

References

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