Exact Algorithms for Linear Matrix Inequalities

Mohab SAFEY EL DIN
Université Pierre et Marie Curie, France

Didier HENRION
LAAS CNRS, Univ. Toulouse, France

Simone NALDI
Fields Institute, Univ. Toronto, Canada

Mots-clefs : Polynomial Optimization, Linear Matrix Inequalities, Symbolic Computation.

Let $A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n$ be a linear matrix, or pencil, generated by given symmetric matrices $A_0, A_1, \ldots, A_n$ of size m with rational entries. The set of real vectors x such that the pencil is positive semidefinite is a convex semi-algebraic set called spectrahedron, described by a linear matrix inequality (LMI). We design an exact algorithm that, up to genericity assumptions on the input matrices, computes an exact algebraic representation of at least one point in the spectrahedron, or decides that it is empty. The algorithm does not assume the existence of an interior point, and the computed point minimizes the rank of the pencil on the spectrahedron. The degree d of the algebraic representation of the point coincides experimentally with the algebraic degree of a generic semidefinite program associated to the pencil. We provide explicit bounds for the complexity of our algorithm, proving that the maximum number of arithmetic operations that are performed is essentially quadratic in a multilinear Bézout bound of d. When the size m of the pencil is fixed, such a bound, and hence the complexity, is polynomial in n, the number of variables. We conclude by providing results of experiments showing significant practical improvements with respect to state-of-the-art computer algebra algorithms.