Mechanism Design and Allocation Algorithms for Network Markets
with piece-wise linear costs and quadratic externalities

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Motivation

- Many countries are undertaking changes in their electricity market regulation.
- Previous works show how regulation mechanisms allow the producers to charge significantly more than their marginal prices.
- **Question raised:** What could be done about this?
1. The two agent problem
   - A simple auction
   - What if we put some frictions in the model?
   - Some previous results
   - Mechanism design

2. General setting

3. Standard allocation
   - The optimization problem
   - Fixed point
   - Regularity

4. Mechanism design
   - Result
   - Discussion
The Two Agent Problem
A simple auction

- On one hand two electricity producers (AKA agents $a_1$ and $a_2$) have a marginal production cost $c_1$ and $c_2$ respectively.
- On the other hand, a central operator (AKA principal) needs to buy two units of electricity.
- Agent $i = 1 .. 2$ bids a marginal price $b_i$.
- The principal chose the lowest bid.
- Nash equilibrium in pure strategy and complete information for symmetric agents: bid $c$, earn 0.
What if we put some frictions in the model?

When a quantity $h$ of electricity is sent from node 1 to node 2 we lose $rh^2$ in the process.

The principal solves

$$\begin{align*}
\text{minimize } & c_1 q_1 + c_2 q_2 \\
\text{subject to: } & q_i - h_i + h_{-i} \geq r\frac{h_i^2}{2} + h_{-i}^2 + d \quad \text{for } i = 1, 2 \\
& q_i, h_i \geq 0 \quad \text{for } i = 1, 2
\end{align*}$$
Some already known facts

Allocation solution

\[ F(x, y) = d + \frac{1}{2r} \left( \frac{x - y}{x + y} \right)^2 - \frac{1}{r} \left( \frac{x - y}{x + y} \right) \quad \tilde{q} = 2 \left[ \frac{1 - \sqrt{1 - 2dr}}{r} \right] \]

Then

\[ q_i(c_i, c_{-i}) = \begin{cases} 
F(c_i, c_{-i}) & \text{if } F(c_i, c_{-i}) \geq 0 \text{ and } F(c_{-i}, c_i) \geq 0 \\
\tilde{q} & \text{if } F(c_{-i}, c_i) < 0 \text{ and } F(c_i, c_{-i}) \geq 0 \\
0 & \text{if } F(c_i, c_{-i}) < 0 \text{ and } F(c_{-i}, c_i) \geq 0 
\end{cases} \]

Market power from the quadratic externalities

\[ b^* = \frac{c}{1 - 2dr} \]
Theorem (Revelation Principle)

To any Bayesian Nash equilibrium of a game of incomplete information, there exists a payoff-equivalent direct revelation mechanism that has an equilibrium where the players truthfully report their types.

- Idea: change the principal behaviors
- Tool: the revelation principle
- So: we perform an optimization over the truthful direct mechanism (see general case later)
Proposition (Under some hypothesis on $f$)

If in a mechanism $(\hat{q}, \hat{h}, \hat{x})$ the assignment function $(\hat{q}, \hat{h})$ solves

$$\min_{q,h} \int_C \sum_{i=1,2} q_i(c)\left[c_i + \frac{F_i(c_i)}{f_i(c_i)}\right]f(c)dc$$

subject to the allocations constraints and the payment function $\hat{x}$ satisfies

$$\hat{x}_i(c) = \hat{q}_i(c)c_i + \int_{c_i}^{\bar{c}_i} q_i(s,c_{-i})ds$$

then $(\hat{q}, \hat{h}, \hat{x})$ is an optimal mechanism.
Figure: The social costs for the standard mechanism and the optimal mechanism.
Strategies

Figure: The bids and costs for different settings

Bids for $a=2$

- Standard bid
- Honesty
- Optimal Cost
At each node $i$:

- There is a fixed demand for electricity $d_i$
- There is an electricity producer whose production cost is piecewise-linear of slopes $(c_i^0, \ldots, c_i^n)$ such that for a production level between $k\bar{q}$ and $(k+1)\bar{q}$, the marginal cost is $c_i^k$
- $c_i$ is unknown, but we have a probability distribution $f_i(c_i)$ on it
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- The nodes are connected by edges
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Objective for the operator (ISO): To produce enough electricity to meet demand while minimizing the total cost
The Standard Allocation
Optimization problem

Problem

\[
\begin{align*}
\text{minimize} \quad & \sum_{i=1}^{n} \sum_{j=1}^{N} q_i^j c_i^j \\
\text{subject to} \quad & \forall i \in I : \sum_{j=1}^{N} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& \forall (i, i') \in E : h_{i,i'} \geq 0 \\
& \forall i \in I, j \in J : q_i^j \geq 0 \\
& \forall i \in I, j \in J : q_i^j \leq \bar{q}.
\end{align*}
\]

(1)
$F_i(\lambda_i, \lambda_{-i}) = d_i + \sum_{i' \in V(i)} \frac{\lambda_{i'} - \lambda_i}{r_{i,i'}(\lambda_i + \lambda_{i'})} + \frac{(\lambda_{i'} - \lambda_i)^2}{2r_{i,i'}(\lambda_i + \lambda_{i'})^2}.$ \hspace{1cm} (2)

$$K_i(\lambda_i) = \begin{cases} [k - 1, k]q & \text{if } \lambda_i = c_{i,k} \\ kq & \text{if } \lambda_i \in ]c_{i,k}, c_{i,k+1}[, k \neq N \\ Nq & \text{if } \lambda_i \in \lambda_i \in ]c_{i,N}, \bar{c}[, \end{cases} \hspace{1cm} (3)$$

**Lemma**

For any $i \in I$ and any $\lambda^{-i} \in [\min_i c_i^1, \max_i c_i^N]^{n-1}$, $\Lambda_i(\lambda_{-i})$ is the unique solution of

$F_i(\Lambda_i(\lambda_{-i}), \lambda_{-i}) \in K_i(\Lambda_i).$ \hspace{1cm} (4)
Theorem

The sequence $(\Lambda^k(\alpha_1^N \ldots \alpha_n^N))_k$ converges to the solution of the dual.
We consider the subset $S$ of $C$ at which at the same time at some nodes, the multiplicator is equal to the marginal cost and the production is a multiple of $\bar{q}$ (i.e. stuck in an angle):

$$S = \{ c \in C^n, q_i(c) = j\bar{q} \text{ and } \lambda_i(c) = c_{j'} \} \quad (5)$$

for some $i \in I, j \in J, j' \in \{j, j + 1\}$. \quad (6)

The set $S$ corresponds to the point of transition between the two possibilities defined by the first order condition.

**Theorem**

The function $q$ is $C^\infty$ on $C^n \setminus S$. 
The Mechanism Design
The expected profit of an agent writes

\[ U_i(c_i, c'_i) = \mathbb{E}_{-i} u_i = X_i(x, c'_i) - \sum_{j \in [1..N]} c^j_i Q^j_i(c'_i). \]

with

\[ Q^j_i(q, c_i) = \mathbb{E}_{-i} \min((q_i(c_i, c_{-i}) - j\bar{q})^+, \bar{q}) \quad \text{and} \quad X_i(x, c_i) = \mathbb{E}_{-i} x_i(c_i, c_{-i}) \]

We denote

\[ \tilde{f}^i_j(c_i, t) = \begin{cases} \frac{f_i(c^{-j}_i, c^j_i)}{f_i(c^{-j}_i, t)} & \text{if } f_i(c^{-j}_i, t) \neq 0 \\ 0 & \text{else} \end{cases} \quad \text{and} \quad K^i_j(c^{-j}_i, t) = \int_0^t \tilde{f}^i_j(c_i, t) \, dc^j_i. \]
The Optimization Problem (P1)

Problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in I} E x_i(c) \\
\text{subject to} & \quad q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i \\
\quad & \quad h_{i,i'}(c) \geq 0 \\
\quad & \quad U_i(c_i, c_i) \geq U_i(c_i, c_i') \\
\quad & \quad U_i(c_i, c_i) \geq 0.
\end{align*}
\]
Problem

\[ \text{minimize } \sum_{i \in I} E x_i(c) \]

subject to.

\[ q_i(c) + \sum_{i' \in V(i)} h_{i,i'}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i \]

\[ h_{i,i'}(c) \geq 0 \]

\[ V_i(c^1, \ldots, c^{j-1}, t_1, c^{j+1}, \ldots, c^N) - V_i(c^1, \ldots, c^{j-1}, t_2, c^{j+1}, \ldots, c^N) = \]

\[ \int_{t_1}^{t_2} Q_i^j(c^1, \ldots, c^{j-1}, s, c^{j+1}, \ldots, c^N) ds \]

\[ (c - c')(Q(c) - Q(c')) \leq 0 \]
Problem

\[
\text{minimize}_{(q,x,h)} \sum_{i \in I} \sum_{j \in J} q^j_i (c_i, c_{-i}) (c^j_i + K^j_i (c_i^{-j}, c^j_i)) \\
\text{subject to} \\
q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i \\
h_{i,i'}(c) \geq 0. \\
x_i(c) = \sum_{j=1}^{N} (c^j_i + K^j_i (c_i^{-j}, c^j_i)) q^j_i(c)
\]
Theorem

Problems 1, 2 and 3 have the same solution.
Discussion

- Stability
- Benchmark algorithm
- Structure of the auction equilibrium
Conclusion

- We presented a framework for the design of wholesale electricity market as well as tools to compare it with a standard auction setting.
- Many related aspects are currently under study.
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