Combinaison de méthodes de contrôle optimal pour le problème d'évitement de conflits aériens

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Joint work with **Sonia CAFIERI** and **Frédéric MESSINE**

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Structure of the presentation

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Framework

- Air traffic management (ATM)
- Literature, optimal control and velocity regulation

Optimal control models and solution methods

- Model
- Decomposition strategy and definition of "zones"
- Direct shooting method on "zone"
- PMP conditions on "prezone" and "postzone"
- Combination of methods and complexity

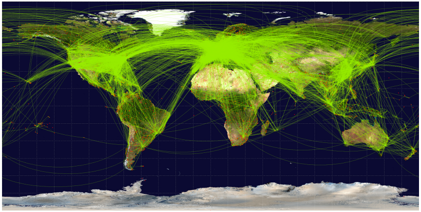
Numerical results

- Benchmarking
- Comparison of solvers
- Comparison of approaches
- Partition (clustering)

Conclusion

- Results and perspectives

Air traffic management



Main civil airways during 24h (2009)

Near real-time air traffic (2015)

Current air traffic (nb. aircraft)

- France............**8 000** a day
- Europe.........**33 000** a day
  (> **10 000 000** a year)

Important air traffic growth

Global traffic (nb. aircraft)

- **×2** 2005 → 2020 (ACARE 2005)
- **+3% ∼ +5%** a year (EEC 2010)
- **+5%** 2014 → 2030 (CORIS 2014)

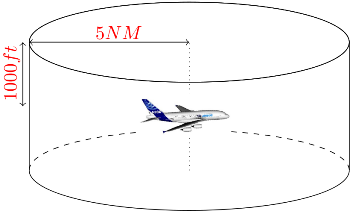
Emerging problems

- **Efficiency**
- **Safety**

Aircraft conflict avoidance

OACI : Two aircraft (i,j) are **in conflict**

$$\text{if} \begin{cases} \| x_i - x_j \| < 5 \text{ NM} \quad (9,26 \text{ km}) \\ \text{and} \\ \| h_i - h_j \| < 1\,000 \text{ ft} \quad (304,8 \text{ m}) \end{cases}$$



Aircraft cylindrical protection volume

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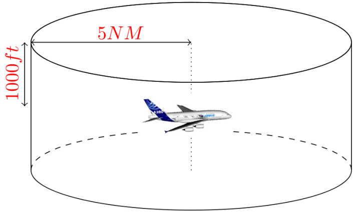

Aircraft cynlindrical protection volume

Conflict resolution

- **Decentralized**
  - Non cooperative
  - Cooperative
- **Centralized** (cooperative)

Avoidance maneuvers

- **heading changes**
- **flight level assignment**
- **velocity changes**

Literature : Optimal control, velocity regulation and ATM

- **Bryson and Denham** (JAM 1962) (Take-off in minimal time)
- **Clements and Ingalls** (OCAM 1999) (CD&R, minimal time, 2 aircraft)
- **Tomlin et al**. (IEEE 2000) (Non-cooperative resolution, 2 aircraft)
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- **Friedman** (TR-B 1987) (Optimal maneuvering instant)
- **Pallottino et al.** (IEEE 2002) (CD&R, relative velocity, MILP)
- **Crück and Lygeros** (ECC 2007) (Reduction of conflict risk, 2 aircraft)
- **Chaloulos et al.** (IRWE 2009) (Decentralized resolution, navigation functions)
- **Vela et al.** (CDC-CCC 2009) (MILP)
- **Alonso-Ayuso et al.** (AOR 2011) (CD&R, MILP, altitude and velocity)
- **Cafieri and Durand** (JOGO 2014) (CD&R, MINLP)
- **Rey et al.** (TS 2015) (Reduction of conflict charge, MILP)
- ...

Structure of the presentation

Optimal control : model ($\mathcal{P}$)

where I is the set of aircraft,
u acceleration command,
v velocity states, x position states

Conflict avoidance by velocity changes
subliminal regulation from **-6%** to **+3%**
around the nominal velocity

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$$\| x_i(t) - x_j(t) \|^2 \geq D^2 \qquad \forall t \in [t_0, t_f] \text{ avoidance}$$

$$\forall (i,j) \in I^2 \text{ and } i < j$$

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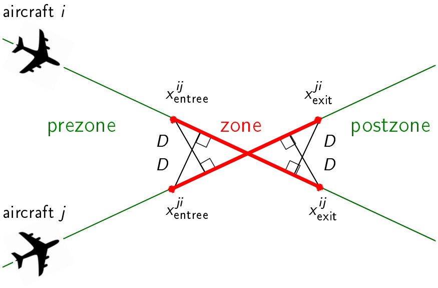
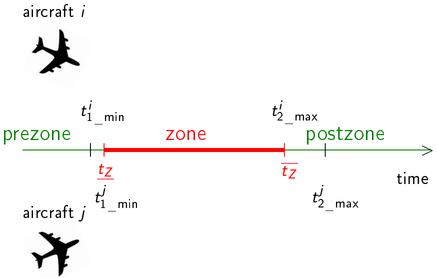
Definition of "*prezone*", "*zone*" and "*postzone*"

Spatial decomposition



Time decomposition



Problem decomposition

Decomposition into ***ZONES***
by taking into account
avoidance conditions

$\Downarrow$

Direct method
in *zone*

&

Necessary conditions due to
Pontryagin's maximum principle
out of *zone*

Time discretization

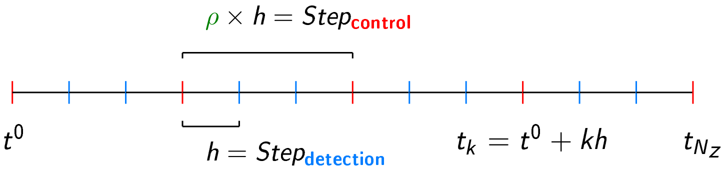
ODE approximations : numerical integrators

$$\mathrm{NUM}_{v_i} : \quad U_i^{\left(\left\lfloor \frac{k}{\rho} \right\rfloor\right)} \longmapsto v_i^{(k+1)} \approx v_i(t_{k+1})$$

$$\mathrm{NUM}_{x_i} : \quad v_i^{(k)} \longmapsto x_i^{(k+1)} \approx x_i(t_{k+1})$$

$$(\forall k \in K = \{0, \ldots, N_Z - 1\}, \forall i \in I)$$

Two time-discretization steps



Example, **Euler** type : $\tilde{x}(t_{k+1}) = x^{(k+1)} \stackrel{\text{def}}{=} x^{(k)} + h_k \tilde{f}\left(t_k, x^{(k)}, U^{(k)}\right)$

Direct shooting method : formulation

$$
(\mathrm{P}_{D^-})\left\{
\begin{array}{ll}
\displaystyle\min_{(x,v,U)\in\mathbb{R}^{nN_Z\left(3+\frac{1}{\rho}\right)}} \rho h \sum_{i\in I}\sum_{l\in M_Z}\left(U_i^{(l)}\right)^2, & \\
\text{subject to} & \\
v_i^{(k+1)} = \mathrm{NUM}_{v_i}\left(U_i^{(\lfloor \frac{k}{\rho}\rfloor)}\right) & \forall k\in K, \forall i\in I,\\
x_i^{(k+1)} = \mathrm{NUM}_{x_i}\left(v_i^{(k)}\right) & \forall k\in K, \forall i\in I,\\
\underline{u_i} \le U_i^{(l)} \le \overline{u_i} & \forall l\in M_Z, \forall i\in I,\\
\underline{v_i} \le v_i^{(k+1)} \le \overline{v_i} & \forall k\in K, \forall i\in I,\\
x_i^{(0)} = x_i^0, \quad v_i^{(0)} = v_i^0, & \forall i\in I,\\
v_i^{(Nz)} = v_i^f, & \forall i\in I,\\
\| x_i^{(k+1)} - x_j^{(k+1)} \|^2 \ \ge\ D^2 & \forall k\in K,\ \forall i<j, (i,j)\in I^2.
\end{array}\right.
$$

$$
(\mathrm{P}_{D_c^-})\left\{
\begin{array}{ll}
\displaystyle\min_{U\in\mathbb{R}^{n\frac{N_Z}{\rho}}} \rho h \sum_{i\in I}\sum_{l\in M_Z}\left(U_i^{(l)}\right)^2, & \\
\text{subject to} & \\
\underline{u_i} \le U_i^{(l)} \le \overline{u_i}, & \forall l\in M_Z,\quad \forall i\in I,\\
\underline{v_i} \le \mathrm{NUM}_{v_i}\left(U_i^{(l)}\right) \le \overline{v_i}, & \forall l\in M_Z,\quad \forall i\in I,\\
x_i^{(0)} = x_i^0 \qquad v_i^{(0)} = v_i^0, & \forall i\in I,\\
\mathrm{NUM}_{v_i}\left(U_i^{(\lfloor \frac{N_Z}{\rho}\rfloor)}\right) = v_i^f, & \forall i\in I,\\
\| \mathrm{NUM}_{x_i}\left(\mathrm{NUM}_{v_i}\left(U_i^{(k+1)}\right)\right) - \mathrm{NUM}_{x_j}\left(\mathrm{NUM}_{v_j}\left(U_j^{(k+1)}\right)\right) \|^2 \ \ge\ D^2 & \\
 & \forall k\in K,\ \forall i<j, (i,j)\in I^2.
\end{array}\right.
$$

synthesis

Pontryagin's maximum principe (without constraints)

$$\begin{cases} \min_{u,x} \quad \mathcal{J}(u(t),x(t)) \overset{\text{def}}{=} \int_{t_0}^{t_f} L\,(u(t),x(t),t)\,dt, & \text{(cost to minimize)} \\ \text{subject to} & \\ \quad \dot{x}(t) = f(u(t),x(t),t), & \text{(dynamical system)} \\ \quad x(t_0) = x^{t_0}, \quad x(t_f) \in \mathcal{C}_f. & \text{(initial and final conditions)} \end{cases}$$

(where $L(.)$ and $f(.)$ are given applications, and $\mathcal{C}_f$ is the set of final states)

$\Longrightarrow$ Introduction to adjoint states : z, one scalar : z_0,
and Hamiltonian : $H(x,z,z_0,u) \overset{\text{def}}{=} zf(x,u) + z_0 L(x,u)$.

Pontryagin's maximum principe : solution $x(.)$ corresponds to the projection of an **extremal** $(x(.),z(.),z_0,u(.))$ satisfying :

$$\dot{x} = \frac{\partial H}{\partial z},\ \dot{z} = -\frac{\partial H}{\partial x},\ H(x,z,z_0,u) = max_{w\in U_{ad}} H(x,z,z_0,w).$$

PMP conditions on "postzone"

Hypothesis

- Constraints on control or states are inactived.
- Extremal is normal.

Solve independently, for each aircraft i,

$$(\mathcal{P}_i)\begin{cases} \min\limits_{u_i} \quad \displaystyle\int_{\overline{t_Z}}^{t_f} u_i^2(t)dt, \\ \text{subject to} \\ \qquad \dot{v}_i(t) = u_i(t) \qquad \forall t \in [\overline{t_Z}, t_f], \\ \qquad \dot{x}_i(t) = v_i(t)d_i \qquad \forall t \in [\overline{t_Z}, t_f], \\ \qquad x_i(\overline{t_Z}) = x_i^{\overline{t_Z}}, \qquad v_i(\overline{t_Z}) = v_i^{\overline{t_Z}}, \\ \qquad x_i(t_f) \quad \text{free}, \qquad v_i(t_f) = v_i^{t_f}. \end{cases}$$

Existence of extremal : Filippov's theorem (1962)

PMP conditions on "postzone"

For all $(x_i, v_i, z_{1,i}, z_{2,i}, z_{0,i}, u_i)$, the Hamiltonian of problem $(\mathcal{P}_i)$ is:

$$H_i\left(x_i(t), v_i(t), z_{1,i}(t), z_{2,i}(t), z_{0,i}, u_i(t)\right) = z_{1,i}(t)v_i(t)d_i + z_{2,i}(t)u_i(t) + z_{0,i}u_i^2(t).$$

We obtain, for almost all instant t in $[\overline{t_Z}, t_f]$:

$$\begin{cases} \dot{x}_i(t) & = v_i(t)d_i, \\ \dot{v}_i(t) & = \dfrac{z_{2,i}(t)}{2}, \\ \dot{z}_{1,i}(t) & = 0_{\mathbb{R}^2}, \\ \dot{z}_{2,i}(t) & = -z_{1,i}(t)d_i. \end{cases}$$

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$$\begin{cases} u_i(t) = \dfrac{v_i^{t_f} - v_i^{\overline{t_Z}}}{t_f - \overline{t_Z}}, \\ v_i(t) = \dfrac{v_i^{t_f} - v_i^{\overline{t_Z}}}{t_f - \overline{t_Z}}(t - t_f) + v_i^{t_f}, \\ x_i(t) = \dfrac{v_i^{t_f} - v_i^{\overline{t_Z}}}{t_f - \overline{t_Z}} d_i \dfrac{t^2}{2} + \left(v_i^{t_f} - \dfrac{v_i^{t_f} - v_i^{\overline{t_Z}}}{t_f - \overline{t_Z}} t_f\right) d_i t \\ \qquad - \left(\dfrac{v_i^{t_f} - v_i^{\overline{t_Z}}}{t_f - \overline{t_Z}}(\overline{t_Z} - t_f) + v_i^{t_f}\right) d_i \overline{t_Z} + x_i^{\overline{t_Z}}. \end{cases}$$

Result: On $[\overline{t_Z}, t_f]$, the optimal acceleration is constant!

Combination of direct shooting method and PMP conditions on “postzone” : formulation

$$
(\mathrm{P}_{C1}) \begin{cases}
\displaystyle\min_{(x,v,U)\in\mathbb{R}^{nN_Z\left(3+\frac{1}{\rho}\right)}} \sum_{i\in I} h\left(\rho \sum_{l\in M_Z} (U_i^{(l)})^2 + \left(\frac{v_i^f - v_i^{(N_Z)}}{t_f - N_Z h}\right)^2\right), \\
\text{subject to} \\
v_i^{(k+1)} = \mathrm{NUM}_{v_i}\left(U_i^{\left(\left\lfloor \frac{k}{\rho}\right\rfloor\right)}\right) & \forall k\in K, \forall i\in I, \\
x_i^{(k+1)} = \mathrm{NUM}_{x_i}\left(v_i^{(k)}\right) & \forall k\in K, \forall i\in I, \\
\underline{u_i} \leq U_i^{(l)} \leq \overline{u_i} & \forall l\in M_Z, \forall i\in I, \\
\underline{u_i} \leq \dfrac{v_i^f - v_i^{(N_Z)}}{t_f - N_Z h} \leq \overline{u_i} & \forall i\in I, \\
\underline{v_i} \leq v_i^{(k+1)} \leq \overline{v_i} & \forall k\in K, \forall i\in I, \\
x_i^{(0)} = x_i^0, \quad v_i^{(0)} = v_i^0, & \forall i\in I, \\
\| x_i^{(k)} - x_j^{(k)} \|^2 \geq D^2 & \forall k\in\left\{\left\lfloor\frac{t_Z}{h}\right\rfloor, \ldots, \left\lceil\frac{t_Z}{h}\right\rceil\right\}, \forall (i,j)\in\mathcal{R}_p.
\end{cases}
$$

synthesis

PMP conditions on "prezone" : hypothesis

Hypothesis

- Consider at most one instant t_i^c in $[t_0, \underline{t_Z}]$ at which the velocity constraint is active, and such that the solution structure corresponds to :
  1. one non-constraint arc in terms of control or velocity (on $[t_0, t_i^c]$)
  2. one *boundary* arc (active velocity constraint on $[t_i^c, \underline{t_Z}]$)
- Extremal is normal.

For each aircraft i, the values A_i and B_i *non-linearily* depend on the instant t_i^c.

$$\begin{cases} A_i \overset{\text{def}}{=} -12\left(\dfrac{x_i^{\diamondsuit\, t_i^c} - x_i^{\diamondsuit\, t_0}}{d_i^{\diamondsuit}(t_i^c)^3} - \dfrac{v_i^{t_i^c} - v_i^{t_0}}{2(t_i^c)^2}\right) \\ B_i \overset{\text{def}}{=} 6\left(\dfrac{x_i^{\diamondsuit\, t_i^c} - x_i^{\diamondsuit\, t_0}}{d_i^{\diamondsuit}(t_i^c)^2}\right) - 2\left(\dfrac{v_i^{t_i^c} - 2v_i^{t_0}}{t_i^c}\right) \end{cases}$$

PMP conditions on "prezone"

Solve independently, for each aircraft i,

$$(\mathcal{P}'_i)\begin{cases} \min\limits_{u_i} \quad \int_{t_0}^{\underline{t_Z}} u_i^2(t)dt, \\ \text{subject to} \\ \quad \dot{v}_i(t) = u_i(t) \qquad \forall t \in [t_0, \underline{t_Z}], \\ \quad \dot{x}_i(t) = v_i(t)d_i \qquad \forall t \in [t_0, \underline{t_Z}], \\ \quad x_i(t_0) = x_i^{t_0}, \qquad v_i(t_0) = v_i^{t_0}, \\ \quad x_i(\underline{t_Z}) = x_i^{\underline{t_Z}}, \qquad v_i(\underline{t_Z}) = v_i^{\underline{t_Z}}. \end{cases}$$

Existence of extremal Filippov's theorem (1962)

Note: the hamiltonian $(\mathcal{P}'_i)$ correspond to the hamiltonian of $(\mathcal{P}_i)$:
$H_i = z_{1,i}(t)v_i(t)d_i + z_{2,i}(t)u_i(t) + z_{0,i}u_i^2(t)$.

PMP conditions on "prezone"

We obtain, for almost all t in $[t_0, t_i^c]$:

$$
(\mathcal{S}_i')\begin{cases}
u_i(t) = -12\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^3} - \dfrac{v_i^{t_i^c} - v_i^{t_0}}{2(t_i^c)^2}\right) t \\
\qquad + 2\left(3\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^2}\right) - \left(\dfrac{v_i^{t_i^c} - 2v_i^{t_0}}{t_i^c}\right)\right), \\
v_i(t) = -6\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^3} - \dfrac{v_i^{t_i^c} - v_i^{t_0}}{2(t_i^c)^2}\right) t^2 \\
\qquad + 2\left(3\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^2}\right) - \left(\dfrac{v_i^{t_i^c} - 2v_i^{t_0}}{t_i^c}\right)\right) t + v_i^{t_0}, \\
x_i(t) = -2\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^3} - \dfrac{v_i^{t_i^c} - v_i^{t_0}}{2(t_i^c)^2}\right) d_i t^3 \\
\qquad + \left(3\left(\dfrac{x_i^{\diamond t_i^c} - x_i^{\diamond t_0}}{d_i^{\diamond}(t_i^c)^2}\right) - \left(\dfrac{v_i^{t_i^c} - 2v_i^{t_0}}{t_i^c}\right)\right) d_i t^2 + v_i^{t_0} d_i t + x_i^{t_0}.
\end{cases}
$$

Result : On $[t_0, t_i^c]$, the optimal acceleration is linear !

Combination of direct shooting method and PMP on "postzone" and "prezone" : formulation

$$
(\mathrm{P}_{C2})\left\{
\begin{array}{ll}
\displaystyle \min_{(x,v,U,A,B,t^c)\in\mathbb{R}^{n\left(N_Z\left(3+\frac{1}{\rho}\right)+3\right)}} \sum_{i\in I}\left(\frac{1}{3}A_i^2(t_i^c)^3 + A_iB_i(t_i^c)^2 + B_i^2 t_i^c + h\left(\rho\sum_{l\in M_Z}\left(U_i^{(l)}\right)^2 + \left(\frac{v_i^f - v_i^{(N_Z)}}{t_f - N_Z h}\right)^2\right)\right), & \\
\text{subject to} & \\
v_i^{(k+1)} = \mathrm{NUM}_{v_i}\left(U_i^{\left(\left\lfloor \frac{k}{\rho}\right\rfloor\right)}\right) & \forall k\in K, \forall i\in I,\\
x_i^{(k+1)} = \mathrm{NUM}_{x_i}\left(v_i^{(k)}\right) & \forall k\in K, \forall i\in I,\\
\underline{u_i} \le U_i^{(l)} \le \overline{u_i} & \forall l\in M_Z, \forall i\in I,\\
t_0 \le t_i^c \le \underline{t_Z} & \forall i\in I,\\
\underline{u_i} \le B_i \le \overline{u_i} & \forall i\in I,\\
\underline{u_i} \le A_i t_i^c + B_i \le \overline{u_i} & \forall i\in I,\\
\underline{u_i} \le \dfrac{v_i^f - v_i^{(N_Z)}}{t_f - N_Z h} \le \overline{u_i} & \forall i\in I,\\
\underline{v_i} \le v_i^{(k+1)} \le \overline{v_i} & \forall k\in K, \forall i\in I,\\
v_i^{(0)} = \dfrac{1}{2}A_i(t_i^c)^2 + B_i t_i^c + v_i^0 & \forall i\in I,\\
x_i^{(0)} = \dfrac{1}{6}A_i d_i(t_i^c)^3 + B_i d_i(t_i^c)^2 + v_i^0 d_i(\underline{t_Z} - t_i^c) & \forall i\in I,\\
\| x_i^{(k)} - x_j^{(k)} \|^2 \ \ge\ D^2 & \forall k\in\left\{\left\lfloor\dfrac{\underline{t_Z}}{h}\right\rfloor,\dots,\left\lceil\dfrac{\overline{t_Z}}{h}\right\rceil\right\}, \forall (i,j)\in\mathcal{R}_p.
\end{array}\right.
$$

Complexity of the formulations : synthesis

formulations PNL	number of **variables**	number of **constraints**: *equality*	number of **constraints**: *inequality* – bounds	number of **constraints**: *inequality* – avoidance	number of **time steps** N_Z
P_{D-}	$nN_Z\left(3+\frac{1}{\rho}\right)$	$n(3N_Z+1)$	$2nN_Z\left(1+\frac{1}{\rho}\right)$	$\frac{n(n-1)}{2}N_Z$	$\left\lceil\frac{t_f-t_0}{h}\right\rceil$
P_{Dc-}	$n\frac{N_Z}{\rho}$	**n**	$2n\frac{N_Z}{\rho}$	$\frac{n(n-1)}{2}N_Z$	$\left\lceil\frac{t_f-t_0}{h}\right\rceil$
P_D	$nN_Z\left(3+\frac{1}{\rho}\right)$	$n(3N_Z+1)$	$2nN_Z\left(1+\frac{1}{\rho}\right)$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{t_f-t_0}{h}\right\rceil$
P_{D_c}	$n\frac{N_Z}{\rho}$	**n**	$2n\frac{N_Z}{\rho}$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{t_f-t_0}{h}\right\rceil$
P_{C1}	$nN_Z\left(3+\frac{1}{\rho}\right)$	$3nN_Z$	$2n\left(N_Z(1+\frac{1}{\rho})+1\right)$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{\overline{t_Z}-t_0}{h}\right\rceil$
P_{C1_c}	$n\frac{N_Z}{\rho}$	**0**	$2n\left(\frac{N_Z}{\rho}+1\right)$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{\overline{t_Z}-t_0}{h}\right\rceil$
P_{C2}	$n\left(N_Z(3+\frac{1}{\rho})+3\right)$	$3nN_Z$	$2n\left(N_Z(1+\frac{1}{\rho})+4\right)$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor$
P_{C2_c}	$n\left(\frac{N_Z}{\rho}+3\right)$	**0**	$2n\left(2\frac{N_Z}{\rho}+4\right)$	$\lvert\mathcal{R}_p\rvert\left(1+\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor\right)$	$\left\lceil\frac{\overline{t_Z}}{h}\right\rceil-\left\lfloor\frac{\underline{t_Z}}{h}\right\rfloor$

where n number of aircraft, $\underline{t_Z}$ and $\overline{t_Z}$ instants which define the "zone",
$\mid\mathcal{R}_p\mid$ number of "rencontres potentielles", $\rho=\frac{\text{time step of command}}{\text{time step of detection}}$

Complexity of formulations : examples

3 avions, $\mid \mathcal{R}_p \mid = 2$, $\underline{t_Z} = 100 \times h$ et $\overline{t_Z} = 140 \times h$ (zone $\approx 17,08\%$ time window)

formu-lations PNL	number de **variables**	number of **constraints** *equalities*	*inequalities* bounds	avoidance	number of **time step** N_Z
P_{D^-}	2340	2163	1800	720	240
$P_{D_c^-}$	180	3	360	720	240
P_D	2340	2163	1800	82	240
P_{D_c}	180	3	360	82	240
P_{C1}	1365	1260	1056	82	140
P_{C1_c}	105	0	216	82	140
P_{C2}	399	360	324	82	40
P_{C2_c}	39	0	144	82	40

Complexity of formulations : examples

3 avions, $|\mathcal{R}_p| = 2$, $\underline{t_Z} = 100 \times h$ et $\overline{t_Z} = 140 \times h$ (zone $\approx 17,08\%$ time window)

formu-lations PNL	number de **variables**	number of **constraints** *equalities*	*inequalities* bounds	*inequalities* avoidance	number of **time step** N_Z
P_{D^-}	2340	2163	1800	720	240
$P_{D_c^-}$	180	3	360	720	240
P_D	2340	2163	1800	82	240
P_{D_c}	180	3	360	82	240
P_{C1}	1365	1260	1056	82	140
P_{C1_c}	105	0	216	82	140
P_{C2}	399	360	324	82	40
P_{C2_c}	39	0	144	82	40

8 aircraft, $|\mathcal{R}_p| = 12$, $\underline{t_Z} = 72 \times h$ and $\overline{t_Z} = 180 \times h$ (zone $\approx 45,42\%$ time window)

formu-lations PNL	number of **variables**	number of **constraints** *equalities*	*inequalities* bounds	*inequalities* avoidance	number of **time step** N_Z
P_{D^-}	6240	5768	4800	6720	240
$P_{D_c^-}$	480	8	960	6720	240
P_D	6240	5768	4800	1308	240
P_{D_c}	480	8	960	1308	240
P_{C1}	4680	4320	3616	1308	180
P_{C1_c}	360	0	736	1308	180
P_{C2}	2832	2592	2224	1308	108
P_{C2_c}	240	0	928	1308	108

Structure of the presentation

Velocity constraints : subliminal regulation (from −6% to +3%)

2 aircraft conflict

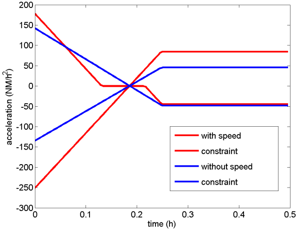
4 aircraft conflict

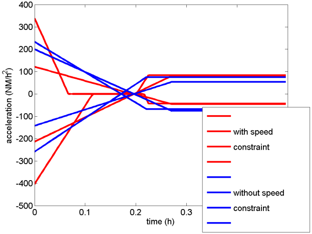
Solution curves of **acceleration**

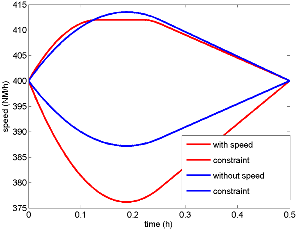
WITHOUT velocity constraints

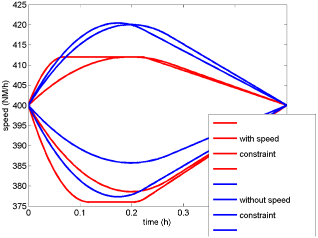
WITH velocity constraints

Solution curves of **velocity**







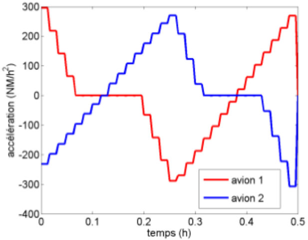


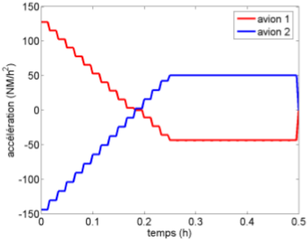
Terminal conditions : with *fixed or free* final positions

2 aircraft conflict

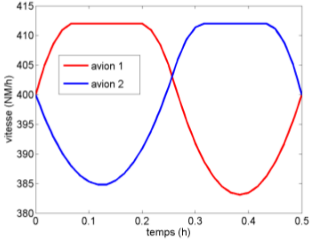
FIXED final positions | **FREE final positions**

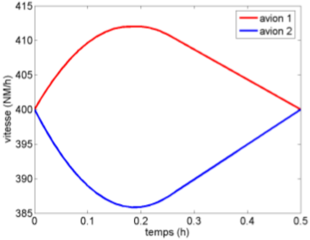
Solution curves of **acceleration**





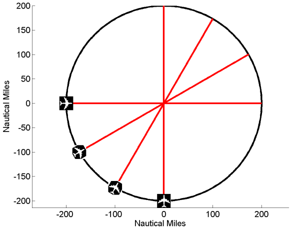
Solution curves of **velocity**

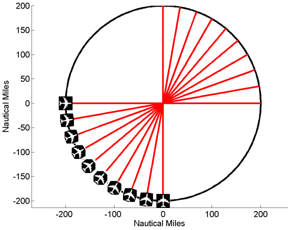




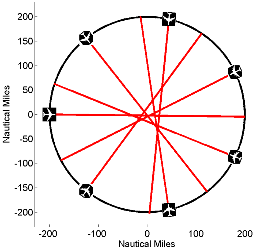
Instances : *circle without/with deviation*

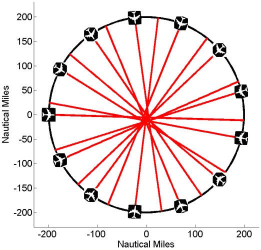
Circle
WITHOUT
deviation





Circle
WITH
deviation

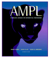




Experimental framework

MATLAB MathWorks

⇒ for the two NLP formulations : (P) and (P_c)

AMPL 

⇒ only for the formulation (P)

NLP solvers :

- MATLAB/`fmincon` – 'sqp' solver
- MATLAB/`fmincon` – 'interior-point' solver
- AMPL/SNOPT (active set method, Gill et al. 2002)
- AMPL/IPOPT (interior point method, Wächter et Biegler 2006)

Instances : data

- For each aircraft, nominal velocity : $v_i^0 = 447$ NM/h (≈ 830 km/h).
- Bounds on velocity : $[v_i^0\text{-}6\%v_i^0, v_i^0+3\%v_i^0]$ (ERASMUS project).
- Bounds on accelerations : $-\underline{u_i} = \overline{u_i} = 4000$ NM/h^2 (BADA manual).
- Time window : $[t_0, t_f] = 1$h
- Time discretization steps : $15''/1'$ (detection/control)

ID	nb. aircraft	nb. conflicts	*zone time* (in %)	(P_{C1}) form. var.	(P_{C1}) form. constr.	(P_{C1_c}) form. var.	(P_{C1_c}) form. constr.
pb41	7	3	15.77	2821	5593	217	2555
pb42	8	4	15.77	3536	7368	272	3560
pb43	9	6	21.99	4095	8820	315	4410
pb44	6	3	4.56	2574	4641	198	1869
pb45	8	4	4.56	3432	6340	264	2644
pb46	5	5	26.97	3120	5910	240	2550
pb47	2	1	6.08	546	936	42	348

Comparison of solvers

Tests run on : CPU 2.53GHz/4Go for the solvers using AMPL
and CPU 3.2GHz/4Go for the solvers using MATLAB.

	(P_{C1}) with AMPL/SNOPT			(P_{C1}) with AMPL/IPOPT			(P_{C1_c}) with MATLAB/fmincon/'ip'		
	nb it.	objective	time	nb it.	objective	time	nb it.	objective	time
pb41	3100	4339.58	14.477	276	**4339.58**	**5.413**	55	5824.4	43.415
pb42	4056	5180.19	19.048	245	**5180.19**	**10.621**	–	–	$> 10'$
pb43	–	–	$> 10'$	556	**39742.2**	**52.869**	–	–	$> 10'$
pb44	2804	**2882.6**	**14.726**	1175	2882.6	56.474	48	3576.9	22.979
pb45	5384	**3843.47**	**65.193**	1565	3843.47	116.189	45	4395.2	41.668
pb46	3924	**4553.92**	**9.126**	394	8282.67	19.701	130	22235	127.765
pb47	–	–	$> 10'$	91	**1631.47**	**0.484**	100	3152.6	3.947

Note : IPOPT seems to be more robust than SNOPT to solve this instances.
The minimizing functions obtained by MATLAB have higher cost
than the minimizing functions obtained by IPOPT and SNOPT.

Instances : data

Names pb.	nb **aircraft**	nb **conflict**	nb **rencontres**	time in "*zone*" **(%)**	**velocity** (NM/h) 400	447	[400; 450]	**velocity regulation** (%) **[-6 ; 3]**	[-12 ; 6]	[-10 ; 10]	**sum of conflict times** (seconds)
pb01	**3**	**2**	**3**	**13**	✓					✓	**61.0**
pb02	3	2	3	16	✓					✓	50.5
pb03	3	2	2	11	✓			✓			73.3
pb04	4	3	3	16			✓			✓	150.0
pb05	4	3	3	16			✓		✓		102.2
pb20	10	5	5	31		✓			✓		232.0
pb21	**10**	**5**	**5**	**31**		✓				✓	**232.0**
pb22	10	5	5	35	✓				✓		259.2
pb23	11	6	17	45	✓			✓			521.1
pb24	11	5	7	54	✓			✓			259.1
pb25	11	6	15	51			✓		✓		320.4
pb26	12	6	12	32	✓			✓			643.0
pb27	12	6	15	46		✓		✓			472.2
pb28	12	6	12	29		✓		✓			575.4
pb29	13	6	14	59		✓		✓			418.3
pb30	**13**	**9**	**14**	**67**			✓	✓			**1294.0**

Note : Indicators of air traffic complexity : ***sum of conflict times*** and percentage of "***zone time***" (for our approach).

Direct shooting method and PMP conditions on "postzone"

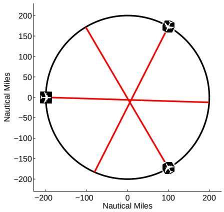
Names and nb. aircraft	nb var.	nb contr.	nb it.	nb constr.	objective value	time (seconds)
pb01 - 3	**2358**	**4698**	**12**	**16**	**77.774**	**0.272**
pb02 - 3	2358	4698	21	78	144.714	0.476
pb03 - 3	2358	4698	20	107	907.372	0.472
pb04 - 4	3144	6744	33	145	1068.120	1.628
pb05 - 4	3144	6744	75	479	644.613	3.388
pb20 - 10	7860	24060	87	843	6099.380	124.072
pb21 - 10	**7860**	**24060**	**372**	**4403**	**6099.380**	**533.325**
pb22 - 10	7860	24060	87	836	4893.630	123.508
pb23 - 11	8646	27786	215	1208	5855.790	420.366
pb24 - 11	8646	27786	178	1565	1604.610	394.725
pb25 - 11	8646	27786	115	895	2665.110	256.476
pb26 - 12	9432	31752	183	1237	2400.130	594.361
pb27 - 12	9432	31752	357	2665	6102.880	1108.420
pb28 - 12	9432	31752	235	1747	3051.040	832.024
pb29 - 13	10218	35958	242	1880	4906.200	986.822
pb30 - 13	**10218**	**35958**	–	–	–	$>$ **1200.000**

Direct shooting method and PMP conditions on "postzone"

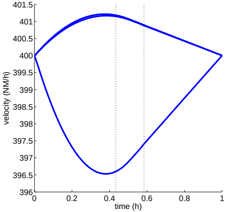
Names and nb. aircraft	nb var.	nb contr.	nb it.	nb constr.	objective value	time (seconds)	nb var.	nb contr.	nb it.	nb eval.	objective value	time (seconds)	benefit time (%)
pb01 - 3	**2358**	**4698**	**12**	**16**	**77.774**	**0.272**	**1383**	**2421**	**11**	**14**	**77.774**	**0.096**	**64.706**
pb02 - 3	2358	4698	21	78	144.714	0.476	1383	2439	18	53	144.714	0.152	68.067
pb03 - 3	2358	4698	20	107	907.372	0.472	1383	2403	20	80	907.372	0.148	68.644
pb04 - 4	3144	6744	33	145	1068.120	1.628	1896	3420	28	95	1068.120	0.416	74.447
pb05 - 4	3144	6744	75	479	644.613	3.388	1792	3244	31	99	644.613	0.356	89.492
pb20 - 10	7860	24060	87	843	6099.380	124.072	4740	11460	60	478	6099.380	25.306	79.604
pb21 - 10	**7860**	**24060**	**372**	**4403**	**6099.380**	**533.325**	**4740**	**11415**	**35**	**207**	**6099.380**	**15.377**	**97.117**
pb22 - 10	7860	24060	87	836	4893.630	123.508	5390	12965	61	475	4893.630	33.706	72.709
pb23 - 11	8646	27786	215	1208	5855.790	420.366	5786	15763	110	346	5855.790	87.793	79.115
pb24 - 11	8646	27786	178	1565	1604.610	394.725	6930	18964	49	341	1604.610	60.732	84.614
pb25 - 11	8646	27786	115	895	2665.110	256.476	6358	17556	63	394	2665.110	79.165	69.134
pb26 - 12	9432	31752	183	1237	2400.130	594.361	6000	15348	99	463	2400.130	72.744	87.761
pb27 - 12	9432	31752	357	2665	6102.880	1108.420	5844	17196	167	1190	6102.880	244.123	77.976
pb28 - 12	9432	31752	235	1747	3051.040	832.024	5376	13698	98	471	3051.040	81.437	90.212
pb29 - 13	10218	35958	242	1880	4906.200	986.822	8528	27131	177	1251	4906.200	451.824	54.214
pb30 - 13	**10218**	**35958**	–	–	–	> 1200.000	**7683**	**24219**	**164**	**974**	**3011.190**	**326.672**	–

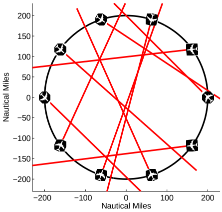
Note: Reduction of the number of variables and constraints, benefit in terms of computing time.

Examples : trajectories & velocities

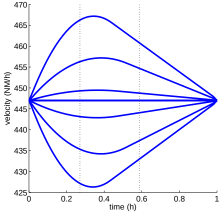


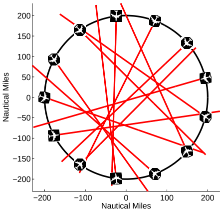
Conflict avoidance with **3** aircraft (pb01, time= 0′00″096)



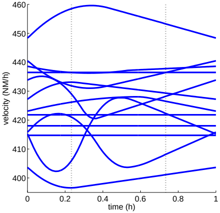


Conflict avoidance with **10** aircraft (pb21, time= 0′15″377)





Conflict avoidance with **13** aircraft (pb30, time= 5′26″672)



Partition (clustering) : data & results

Cluster of aircraft
= graph connected component :

1. vertices : aircraft
2. edges : which link potential conflicts.

$\Rightarrow$ Method : heuristic

Partition (clustering) : data & results

Cluster of aircraft (our approach)
= graph connected component :

1. vertices : aircraft
2. edges : which link ~~potential conflits.~~ **potential rencontres.**

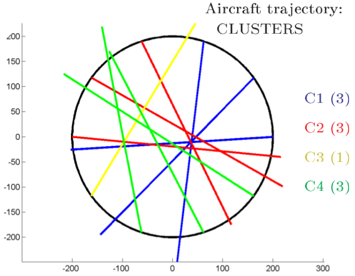
$\Rightarrow$ Method : ~~heuristic~~ **exact**

	nb aircraft	nb **rencontres**	nb conflicts	cardinality of **clusters**
pb31	10	**11**	4	1 − 1 − 2 − 1 − 5
pb32	10	**10**	1	1 − 4 − 2 − 3
pb33	10	**6**	3	1 − 2 − 2 − 3 − 2
pb34	10	**14**	2	5 − 4 − 1
pb35	10	**8**	3	3 − 3 − 1 − 3
pb36	8	**4**	4	2 − 2 − 2 − 2
pb37	10	**5**	5	2 − 2 − 2 − 2 − 2
pb38	10	**16**	10	5 − 5

	(P_D) WITHOUT decomposition					(P_{C1}) **WITH decomposition**					benefit (in %)				
	var.	contr.	it.	eval.	time	var.*	contr.*	it.*	eval.*	time	var.	contr.	it.	eval.	time
pb31	7750	13225	–	–	$> 20'$	3875	5185	646	1787	28.68	50	60.79	–	–	–
pb32	7750	14620	–	–	$> 20'$	3100	3760	534	2823	12.16	60	74.28	–	–	–
pb33	7620	12295	1045	1897	461.57	2965	3195	325	349	5.74	61.09	74.01	68.90	81.60	98.76
pb34	6840	12295	–	–	$> 20'$	4104	5781	299	1046	22.27	40	52.98	–	–	–
pb35	7750	13945	–	–	$> 20'$	2325	2487	290	2075	4.77	70	82.17	–	–	–
pb36	3392	3660	1421	2399	34.02	848	801	258	286	3.11	75	78.11	81.84	88.08	90.86
pb37	4240	4765	–	–	$> 20'$	848	801	268	278	4.23	80	82.2	–	–	–
pb38	5930	9610	–	–	$> 20'$	3095	3515	460	586	15.98	47.81	63.42	–	–	–

Note : Reduction of nb. of variables and constraints, benefit of computing time

Partition (clustering) : solutions

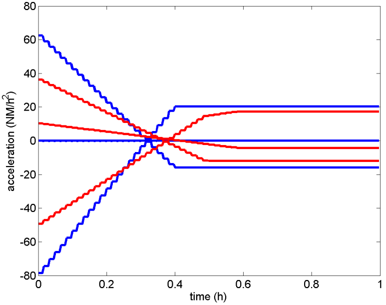


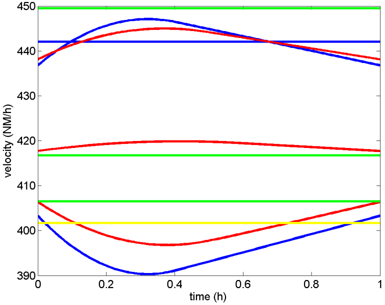
Conflict avoidance with **10** aircraft

(←) trajectories and clusters of aircraft

(↙) curves of optimal **acceleration**

(↘) curves of optimal **velocity**





Structure of the presentation

Conclusion & comments

Summary of work

Model and solution approaches through **optimal control**

- **Decomposition** of the problem : definition of "*zones*"
- **Combination** of shooting methods
- **Implementation** and comparison of numerical envieronments

Summary of results

Advantages

- **Continuity of velocity solutions**
- **Subliminal velocity variations**
- **Smooth regulation** (compatible with current ATM system)
- **Information of states** (along the whole time window)
- **Reasonable computing time** (for small number of aircraft)

Limits

- No garantee of **feasibility** (using only small speed range)
- No taking into account **uncertainties**

Perspectives

- Numerical resolution of the formulation "prezone" using PMP conditions
- Study of the optimal solution structure on "prezone"
- Study of the partition (clustering) of aircraft for this problem
- Study of others criteria for the aircraft conflict avoidance model
- ...

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Thanks for your attention / Questions

THANKS

Questions

Picture snapped from "place du capitole"