# Mechanism Design and Allocation Algorithms for Network Markets

with piece-wise linear costs and quadratic externalities

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# Motivation

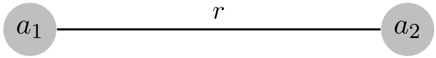
- Many countries are undertaking changes in their electricity market regulation.
- Previous works show how regulation mechanisms allow the producers to charge significantly more than their marginal prices.
- Question raised: What could be done about this ?

# The Two Agent Problem

# A simple auction

- On one hand two electricity producers (AKA agents $a_1$ and $a_2$) have a marginal production cost $c_1$ and $c_2$ respectively.
- On the other hand, a central operator (AKA principal) need to by two units of electricity.
- Agent $i = 1..2$ bids a marginal price $b_i$.
- The principal chose the lowest bid.
- Nash equilibrium in pur strategy and complete information for symmetric agents: bid $c$, earn 0.

# What if we put some frictions in the model ?




When a quantity $h$ of electricity is sent from node 1 to node 2 we loose $rh^2$ in the process.

**The principal solves**

$$\underset{q,h}{\text{minimize}}\ c_1 q_1 + c_2 q_2$$

subject to:

$$q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \text{ for } i = 1, 2$$

$$q_i, h_i \geq 0 \text{ for } i = 1, 2$$

♣

# Some already known facts

## Allocation solution

$$F(x,y) = d + \frac{1}{2r}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{r}\left(\frac{x-y}{x+y}\right) \qquad \tilde{q} = 2\left[\frac{1-\sqrt{1-2dr}}{r}\right]$$

Then

$$q_i(c_i, c_{-i}) = \begin{cases} F(c_i, c_{-i}) & \text{if } F(c_i, c_{-i}) \geq 0 \text{ and } F(c_{-i}, c_i) \geq 0 \\ \tilde{q} & \text{if } F(c_{-i}, c_i) < 0 \text{ and } F(c_i, c_{-i}) \geq 0 \\ 0 & \text{if } F(c_i, c_{-i}) < 0 \text{ and } F(c_{-i}, c_i) \geq 0 \end{cases}$$

## Market power from the quadratic externalities

$b^* = \frac{c}{1-2dr}$

**Theorem (Revelation Principle)**

*To any Bayesian Nash equilibrium of a game of incomplete information, there exists a payoff-equivalent direct revelation mechanism that has an equilibrium where the players truthfully report their types.*

- Idea: change the principal behaviors
- Tool: the revelation principle
- So: we perform an optimization over the truthful direct mechanism (see general case later)

**Proposition (Under some hypothesis on $f$)**

*If in a mechanism $(\hat{q}, \hat{h}, \hat{x})$ the assignment function $(\hat{q}, \hat{h})$ solves*

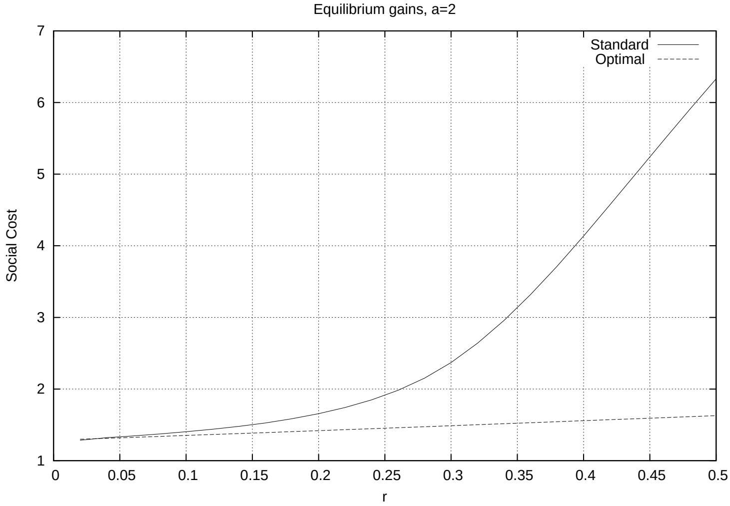
$$\min_{q,h} \int_C \sum_{i=1,2} q_i(c)[c_i + \frac{F_i(c_i)}{f_i(c_i)}]f(c)dc$$

*subject to the allocations constraints and the payment function $\hat{x}$ satisfies*

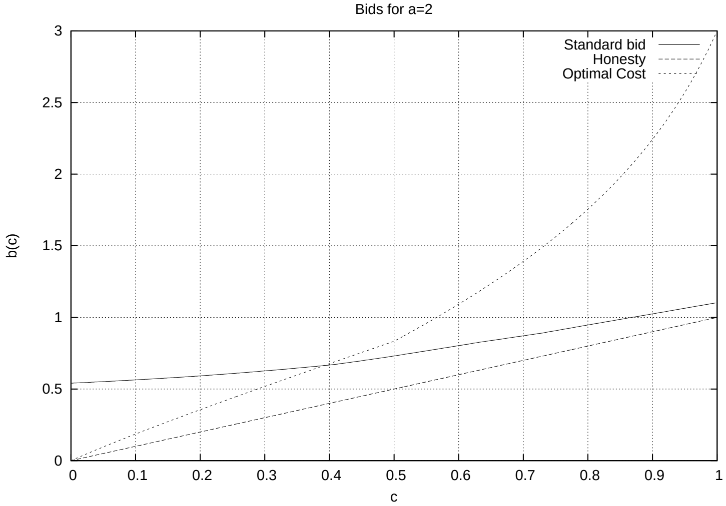
$$\hat{x}_i(c) = \hat{q}_i(c)c_i + \int_{c_i}^{\bar{c}_i} q_i(s, c_{-i})ds$$

*then $(\hat{q}, \hat{h}, \hat{x})$ is an optimal mechanism.*

# Benchmark

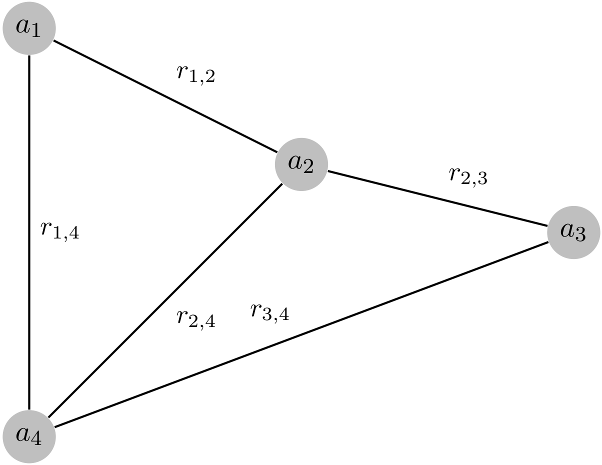

Equilibrium gains, a=2
Standard
Optimal
Social Cost
r
1
2
3
4
5
6
7
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

# Strategies


Bids for a=2
Standard bid
Honesty
Optimal Cost
b(c)
c
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

# General Setting

# Example of Network



## The Setting

- At each node $i$:
  - There is a fixed demand for electricity $d_i$
  - There is an electricity producer whose production cost is piecewise-linear of slopes $(c_i^0, \ldots, c_i^n)$ such that for a production level between $k\bar{q}$ and $(k+1)\bar{q}$, the marginal cost is $c_i^k$
  - $c_i$ is unknown, but we have a probability distribution $f_i(c_i)$ on it

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- The nodes are connected by edges
- When a quantity $h_{i,j}$ of electricity is sent from node $i$ to node $j$ we loose, $r_{i,j}h_{i,j}^2$ in the process
- Objective for the operator (ISO): To produce enough electricity to meet demand while minimizing the total cost

♣

# The Standard Allocation

# Optimization problem

## Problem

$$
\begin{aligned}
\underset{(q,h)}{minimize} \quad & \sum_{i=1}^{n} \sum_{j=1}^{N} q_i^j c_i^j \\
subject\ to \quad & \forall i \in I : \sum_{j=1}^{N} q_i^j + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \\
& \forall (i, i') \in E : h_{i,i'} \geq 0 \\
& \forall i \in I, j \in J : q_i^j \geq 0 \\
& \forall i \in I, j \in J : q_i^j \leq \bar{q}.
\end{aligned}
\tag{1}
$$

# Fixed-Point

$$F_i(\lambda_i, \lambda_{-i}) = d_i + \sum_{i' \in V(i)} \frac{\lambda_{i'} - \lambda_i}{r_{i,i'}(\lambda_i + \lambda_{i'})} + \frac{(\lambda_{i'} - \lambda_i)^2}{2r_{i,i'}(\lambda_i + \lambda_{i'})^2}. \quad (2)$$

$$K_i(\lambda_i) = \begin{cases} [k-1, k]\bar{q} & \text{if } \lambda_i = c_{i,k} \\ k\bar{q} & \text{if } \lambda_i \in ]c_{i,k}, c_{i,k+1}[, k \neq N \\ N\bar{q} & \text{if } \lambda_i \in \lambda_i \in ]c_{i,N}, \bar{c}[, \end{cases} \quad (3)$$

**Lemma**

*For any $i \in I$ and any $\lambda^{-i} \in [\min_i c_i^1, \max_i c_i^N]^{n-1}$, $\Lambda_i(\lambda_{-i})$ is the unique solution of*

$$F_i(\Lambda_i(\lambda_{-i}), \lambda_{-i}) \in K_i(\Lambda_i). \quad (4)$$

# Algorithms

**Theorem**

*The sequence* $(\Lambda^k(c_1^N ... c_n^N))_k$ *converges to the solution of the dual.*

# Regularity

We considere the subset $\mathcal{S}$ of $\boldsymbol{C}$ at which at the same time at some nodes, the multiplicator is equal to the marginal cost and the production is a multiple of $\bar{q}$ (i.e. stuck in an angle):

$$\mathcal{S} = \{c \in \boldsymbol{C}^n, q_i(c) = j\bar{q} \text{ and } \lambda_i(c) = c_{j'} \quad (5)$$

$$\text{for some } i \in I, j \in J, j' \in \{j, j+1\}\}. \quad (6)$$

The set $\mathcal{S}$ corresponds to the point of transition between the two possibilities defined by the first order condition.

**Theorem**

*The function $q$ is $C^\infty$ on $\boldsymbol{C}^n \backslash \mathcal{S}$.*

# The Mechanism Design

# Some definition

The expected profit of an agent writes

$$U_i(c_i, c_i') = \mathbb{E}_{-i} u_i = X_i(x, c_i') - \sum_{j \in [1..N]} c_i^j Q_i^j(c_i').$$

with

$$Q_i^j(q, c_i) = \mathbb{E}_{-i} \min((q_i(c_i, c_{-i}) - j\bar{q})^+, \bar{q}) \quad \text{and} \quad X_i(x, c_i) = \mathbb{E}_{-i} x_i(c_i, c_{-i})$$

We denote

$$\tilde{f}_j^i(c_i, t) = \begin{cases} \frac{f_i(c_i^{-j}, c_i^j)}{f_i(c_i^{-j}, t)} & \text{if } f_i(c_i^{-j}, t) \neq 0 \\ 0 & \text{else} \end{cases} \quad \text{and} \quad K_j^i(c_i^{-j}, t) = \int_0^t \tilde{f}_j^i(c_i, t) dc_i^j.$$

# The Optimization Problem (P1)

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in I} \mathbb{E} x_i(c)$$

$$subject\ to$$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

$$h_{i,i'}(c) \geq 0$$

$$U_i(c_i, c_i) \geq U_i(c_i, c'_i)$$

$$U_i(c_i, c_i) \geq 0.$$

# P2

## Problem

$$\underset{(q,x,h)}{minimize} \sum_{i \in I} \mathbb{E} x_i(c)$$

$$subject\ to.$$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h^2_{i,i'}(c) + h^2_{i',i}(c)}{2} r_{i,i'} \geq d_i$$

$$h_{i,i'}(c) \geq 0$$

$$V_i(c^1, .., c^{j-1}, t_1, c^{j+1} .., c^N) - V_i(c^1, .., c^{j-1}, t_2, c^{j+1} .., c^N) =$$

$$\int_{t_1}^{t_2} Q_i^j(c^1, .., c^{j-1}, s, c^{j+1} .., c^N) \mathrm{d}s$$

$$(c - c').(Q(c) - Q(c')) \leq 0$$

# P3

## Problem

$$\underset{(q,x,h)}{minimize} \mathbb{E} \sum_{i \in I} \sum_{j \in J} q_i^j(c_i, c_{-i})(c_i^j + K_i^j(c_i^{-j}, c_i^j))$$

$$subject\ to$$

$$q_i(c) + \sum_{i' \in V(i)} h_{i',i}(c) - h_{i,i'}(c) - \frac{h_{i,i'}^2(c) + h_{i',i}^2(c)}{2} r_{i,i'} \geq d_i$$

$$h_{i,i'}(c) \geq 0.$$

$$x_i(c) = \sum_{j=1}^{N} (c_i^j + K_i^j(c_i^{-j}, c_i^j)) q_i^j(c)$$

# Result

**Theorem**

*Problems 1, 2 and 3 have the same solution.*

# Discussion

- Stability
- Benchmark algorithm
- Structure of the auction equilibrium

**Conclusion**

- We presented a framework for the design of wholesale electricity market as well as tools to compare it with a standard auction setting.
- Many related aspects are currently under study.

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